Counterterrorism Policy in an Uncertain World

Livio Di Lonardo[*] Tiberiu Dragu[†]

[*]Department of Social and Political Sciences, Bocconi University, Via Guglielmo Roentgen 1, Milan, Italy, 20136, Email: livio.dilonardo@unibocconi.it

[†]Department of Politics, New York University, 19 West 4th St., New York, NY, 10012, Email: tiberiu.dragu@nyu.edu

Abstract

Terrorism prevention is a priority for most democratic polities, however, governments are often unable to precisely assess the threat posed by various terrorist groups. How does a government's uncertainty about terrorists' capacity affect the probability of a terror attack occurring? We develop a game-theoretic model to show that the probability of a successful terrorist attack increases when the government expects to face a terror group with low rather than high capacity for violence. This novel result has important implications for how we evaluate the performance of governments and the security agencies in charge of terrorism prevention.

Keywords: Counterterrorism, Terrorism, Security, Uncertainty.

Preventing terrorist attacks is a daunting challenge for governments. As President Bush once famously said, "we have to be right 100 percent of the time in order to protect this country, and [terrorists] got to be right once". This task is further complicated by the fact that government security agencies do not know the precise extent of the terror threats they face and the potential destructive capacity of terrorist groups. Examples of governments' uncertainty about the resources and capabilities of terror groups abound. In September 2014, one of President Obama's top counterterrorism advisors testified to Congress that ISIS had around 10,000 fighters within its ranks. The following day the CIA presented a new report that estimated the same figure to be between 20,000 and 31,500.[1]

Perhaps the most emblematic example of this phenomenon is what took place after the UK Joint Terrorism Analysis Centre lowered the threat level in May 2005, based on an analysis of the available intelligence that revealed the absence of an imminent threat. Following that analysis, on July 6, 2005, the director general of MI5 Eliza Manningham-Buller reassured 12 senior Labour Party MPs that there was no imminent threat of a terrorist attack in the UK, let alone in London (?). The very next day, on July 7, 2005, the Commissioner of Police of the Metropolis, Ian Blair, publicly stated that the London police had been described "as the envy of the policing world in relation to counterterrorism". Such governmental assurances aside, on July 7, 2005, the UK suffered its deadliest terrorist attack in almost 20 years.

While it is a truism that preventing every terrorist attack may be impossible, the sequence of events in the UK in 2005 and the July 7 terror attack raise the question of what is the relationship between a government's uncertainty about the capacity of a terror group and successful terrorism prevention. Was the 7/7 terrorist attack the result of a miscalculation on the part of the British government or an example of something more systematic about

[1]Peter Baker and Erik Schmitt," Many Missteps in Assessment of ISIS Threat", New York Times, September 29, 2014.

the relationship between a government's uncertainty and failures in terrorism prevention? Answering this question is crucial for our ability to correctly evaluate the performance of government security agencies, and to keep them accountable. What is more, this question is particularly relevant when terrorist groups and governments have ultimate goals that are incompatible to the point that bargaining is not a possibility, and therefore the primary government policy is to attempt to prevent terror attacks.[2]

In what follows, we present a parsimonious model of the interaction between a terrorist group and a government seeking to prevent terrorist attacks. Crucially, the government is uncertain about the terror group's capabilities. The key strategic mechanism at work in our model is that the effectiveness of the terrorists' effort to execute a terror plot is (intuitively) higher when the government's counterterrorism effort is lower. Because of this, the terrorist group lays low precisely when the government ramps up its counterterrorism effort, while the government engages in more counterterrorism activities when the terrorist group exerts more effort in planning terrorist activities.

Our analysis shows that terrorism prevention is more likely to fail when the government expects to face a weak group rather than a strong group, so long as the government's stake in terror prevention is not too low. However, along with a security-enhancing effect, an increase in the likelihood of facing a high-capacity terror group also leads to an increase in the costs of counterterrorism activities. This cost outweighs the benefit of increased security from terror, leaving the government better-off facing a weak group.

Our results have several substantive and policy implications. First, they imply that the occurrence of terrorist attacks when governmental security agencies believe the threat to be low need not be an accident or a mistake, but rather an inherent consequence of the nature

[2]The interactions between governments and terrorist groups animated by strong ideological or (fundamentalist) religious beliefs, such as Al-Qaeda and Aum Shirinko, are likely to fall in this category.

of the strategic interaction between terror groups and governments. In other words, it is not the case that the government misjudges the capability of the terrorists and ex-post would be better off choosing more aggressive counterterrorism measures to tackle the higher-than-expected threat. In fact, when the stakes of terrorism prevention are high, the government is more at risk from terrorism when it *knows* that terrorists have relatively low capacity. This, in turn, helps us understand the policy challenges when a government faces a novel threat from a terror group (of possibly unknown capacity), which in turn responds endogenously to the government's preventive measures. As such, our results contribute to the large literature on prevention of both non-strategic (**???**) and strategic threats (**??**), and they suggest that uncertainty is not necessarily an obstacle to successful terrorism prevention.

Second, our results imply that when bargaining between terrorist groups and target governments is not a viable option, the terrorists benefit from being seen as lacking the capacity to carry out successful violent acts. When the ultimate goals of terrorists and governments are simply incompatible, an image of strength does not allow terrorists to obtain concessions at the bargaining table, but it leads governments to increase their counterterrorism efforts, thereby reducing the terrorists' chances of conducting successful attacks.

Finally, the article contributes to an extensive literature on the political economy of terrorism and counterterrorism. Scholars have analyzed the use of terrorism as a signaling tool (**?**; **?**), the determinants and the effects of repressive policies on security (**?**; **?**; **?**; **?**; **?**) and the political determinants of counterterrorism (**?**; **?**; **?**; **?**; **?**; **?**). We add to this literature by presenting novel results on how the government's uncertainty about terrorists' capacity affects terrorism prevention.

The Model

A government (G) and a terrorist group (T) interact strategically. T seeks to conduct terrorist attacks against G, and G seeks to prevent terrorist acts. G chooses a level of counterterrorism activities, mainly surveillance, $s \in [0,1]$, at a cost given by the function $c_G(s) = \frac{s^2}{2}$, capturing both material (i.e. resources) and immaterial (i.e. curtailment of citizens' civil liberties) costs. T chooses a level of effort in planning terrorist acts, $a \in [0,1]$ at a cost given by the function $c_T(a,\omega) = \frac{\omega a^2}{2}$, with $\omega \in \mathbb{R}_+$ standing for T's capacity. Effort in planning and executing a terrorist act requires investing material resources, e.g. towards the recruitment of operatives and the acquisition of the necessary tools to carry out the attack, and the higher ω is, the harder it is for T to attract funding, recruit operatives, etc.

G and T choose their action simultaneously, implying that T must choose a level of terrorist activities without observing G's choice of surveillance and vice versa. Given the levels of government surveillance and terrorist activities, a terrorist attack happens with probability $f(a,s)$, with $\frac{\partial f(a,s)}{\partial a} > 0$, $\frac{\partial f(a,s)}{\partial s} < 0$, and $\frac{\partial^2 f(a,s)}{\partial a \partial s} < 0$ for all $(a,s) \in (0,1)^2$. The assumptions on the first derivatives are intuitive. The assumption on the cross-partial derivative (i.e. $\frac{\partial^2 f(a,s)}{\partial a \partial s} < 0$) formalizes the intuitive notion that the effectiveness of T's effort to execute a terror plot is higher when G's surveillance is lower. For simplicity we assume that $f(a,s) = a(1-s)$. We denote by $\gamma \in \mathbb{R}_+$ T's benefit and G's cost derived from a successful attack, and we refer to it as the *stakes of terrorism prevention.*

T's capacity can be either high or low. A high-capacity group is characterized by a *lower* cost of conducting terrorist activities, $\omega = \underline{\omega}$, than a low capacity-group, $\omega = \overline{\omega}$, i.e. $\overline{\omega} > \underline{\omega}$. The parameter ω is T's private information, while G's prior that T has high capacity is $Pr(\omega = \underline{\omega}) = \mu \in (0,1)$. Given these specifications, G's utility function is $U_G(a,s) = -\gamma a(1-s) - \frac{s^2}{2}$, and T's utility function is $U_T(a,s,\omega) = \gamma a(1-s) - \frac{\omega a^2}{2}$. We solve for the Bayesian Nash equilibrium of this game, henceforth *equilibrium*.[3]

Analysis

Consider first T's choice. For any level of surveillance s, the optimal level of terrorist activities for a group of capacity $\omega_i \in \{\underline{\omega}, \overline{\omega}\}$ is given by,

$$a_i^*(s) = \frac{\gamma(1-s)}{\omega_i} \tag{1}$$

The best response functions of the high-capacity and low-capacity T are *decreasing* in the level of surveillance chosen by G, implying that from T's standpoint the players' actions are *strategic substitutes*. Consider now G's choice. For any level of terrorist activities a, the optimal level of surveillance is characterized by,

$$s^*(\underline{a}, \overline{a}) = \gamma(\mu\underline{a} + (1-\mu)\overline{a}). \tag{2}$$

G's best response function is *increasing* in the level of effort towards the planning of terrorist activities chosen by the two types of T, meaning that from G's standpoint the players' actions are *strategic complements*. Define $\hat{\omega} \equiv \mu\overline{\omega} + (1-\mu)\underline{\omega}$. By substitution, we obtain

$$s^* = \frac{\gamma^2\hat{\omega}}{\underline{\omega}\overline{\omega} + \gamma^2\hat{\omega}}, \qquad \overline{a}^* = \frac{\gamma\underline{\omega}}{\underline{\omega}\overline{\omega} + \gamma^2\hat{\omega}}, \qquad \underline{a}^* = \frac{\gamma\overline{\omega}}{\underline{\omega}\overline{\omega} + \gamma^2\hat{\omega}}, \tag{3}$$

with a high-capacity T exerting higher effort into terrorist activities than a low-capacity T.

Once we have pinned down the optimal actions that characterize the unique equilibrium of the game, we can assess the impact of μ on the equilibrium actions of G and T.

[3] In the Online Appendix we include proofs to all the formal results along with various robustness exercises. To avoid corner solutions, we assume that $\underline{\omega} > \frac{\gamma\overline{\omega}(1-\gamma\mu)}{\overline{\omega}+\gamma^2(1-\mu)}$.

Proposition 1 *An increase in* μ *leads to (i) an increase in the optimal level of surveillance,* s^**, and (ii) a decrease in the optimal level of terrorist activities,* $\underline{a}^*$ *and* $\overline{a}^*$*.*

As G becomes more likely to face a high-capacity T, it increases surveillance to counter the higher expected level of terrorist activities, with the effect of reducing the probability of a successful attack. T anticipates more aggressive surveillance measures and hence a lower probability of success of a planned attack, and thus decides to lay low. Therefore, an increase in the likelihood of facing a high-capacity terrorist organization generates a *deterrence effect*. When does this deterrence effect translate into an increase in security from terrorism?

Proposition 2 *An increase in* μ *leads to a decrease in the expected probability of a terrorist attack if and only if the stakes of terrorism prevention,* γ*, are large enough.*

An increase in μ has two separate effects on the expected probability of a terrorist attack. First, a direct effect, capturing the increased probability of facing a high-capacity T, which translates in a higher probability of a terrorist attack, since $f(\underline{a}^*, s^*) > f(\overline{a}^*, s^*)$. Second, an indirect effect working through the best responses of T and G, which is nothing but the *deterrence effect* described above, meaning that an increase in μ leads to a lower probability of a successful terrorist attack, conditional on facing a certain type of T.

When the stakes of terrorism prevention are high enough, the deterrence effect dominates the direct effect and an increase in μ leads to a lower expected probability of a terrorist attack. This implies that G is most at risk when it is *commonly known* that T has low capacity. While the conventional wisdom sees a government that does not know the strength of the enemy as being in a disadvantageous position, we show that this need not be the case.

If knowing that T is weak increases the probability of a terrorist attack, is G better off facing enemies with higher capacity? The next proposition answers this question.

Proposition 3 *An increase in* μ *leads to (i) a decrease in the expected utility of both the high-capacity and the low-capacity* T*, and (ii) a decrease in* G*'s ex-ante expected utility.*

When G is more likely to face a high-capacity T, G adopts more aggressive counterterrorism measures while T scales down its effort towards planning and executing an attack. While this reduces the material costs T sustains, it also reduces the probability of a successful attack, with the latter effect dominating the former. This result suggests that *both* types of the group prefer to be known as *weak* by the government. This runs contrary to several results in the existing literature on terrorism, where the terrorists benefit from a reputation of having high capacity when sitting at the bargaining table with the government. However, it is common for terrorist groups and governments to have ultimate goals that are incompatible to the point that bargaining is not a possibility. Terrorist groups animated by strong ideological or religious beliefs are likely to fall in this category, and for these groups our result suggests that an image of strength can be detrimental.

Proposition **??** shows that an increase in μ leads G to increase its anti-terrorism activities. This, in turn, has two effects: it drives up the costs of counterterrorism, and it affects the probability of a terrorist attack. However, even when the stakes of terrorism prevention are high enough and the probability of a terrorist attack decreases in μ, the total effect is negative, and an increase in μ decreases G's equilibrium payoff. This means that even if G is most at risk when it *knows* that T is weak, it is precisely in this case that G's expected utility is the highest. When instead G knows T is strong and the stakes of terror prevention are high, G is more successful at shielding itself from terrorist attacks, but this comes at great cost, in terms of material resources allocated to counterterrorism as well as immaterial resources, such as protection of civil liberties and individual rights.

Conclusion

We have developed a simple model of the interaction between a terrorist group and a government seeking to thwart them. The government is uncertain about the group's capacity to conduct terrorist attacks, which can be high or low. We show that, whenever the stakes of terrorism prevention are sufficiently high, the greater the probability the government faces a low-capacity group, the higher the probability of a successful terrorist attack. While the government is most exposed to the risk of a terrorist attack when facing a low-capacity group, this is the scenario where the government enjoys the highest expected payoff, due to the lower costs expended on counterterrorism.

The substantive merit of our analysis lies in clarifying the relationship between uncertainty and terrorism prevention. Some anecdotal evidence indicates that countries that are targeted by terrorist groups might face the greatest odds of being victims of a terrorist attack precisely when the threat coming from the terrorists is low. We demonstrate how the occurrence of terrorist attacks in times of low threat might not just be the result of possible idiosyncratic mistakes and failures within the security apparatus, but it can, in fact, be due to the structure of the strategic interaction between governments and terrorists.

Additionally, our results offer novel policy insights regarding the pernicious role that uncertainty is believed to play in terrorism prevention. Conventional wisdom indicates that the inability of governments to assess the true strength of their enemies inevitably increases the vulnerability of target countries to terror attacks. However, we show that this narrative is incorrect in certain scenarios. In particular, we show that adding uncertainty in a situation in which the government knows it is facing a weak opponent can reduce, rather than increase, the likelihood of a terrorist attack.

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Appendix A

This appendix contains the following sections. In Appendix A, we prove propositions $1-3$ in the main text, and we also state the results in the main text in a different but equivalent way, so as to further clarify the role of μ in our model. In Appendix B, we solve a version of the model where G moves first, and T moves after having observed G's choice of surveillance. In Appendix C, we solve a version of our model in which G is uncertain about how the terrorists value a successful attack, while there is no uncertainty about T's cost of conducting terrorist activities. In Appendix D, we solve a version of the model in which the occurrence of a terrorist attack does not generate zero-sum payoffs for G and T. In Appendix E, we expand our model to try understanding how governments with different capacity to carry out antiterrorism operations are vulnerable to terrorist attack. In Appendix F, we solve a more general version of our model in which we do not use specific functional forms. Finally, in Appendix G, we show that our main results hold in a dynamic (two-period) game in which the government updates about the type of the opponent they faced given the realized outcome and their own investment.

Appendix A

For convenience, write $\beta \equiv \underline{\omega}\overline{\omega} + \gamma^2\hat{\omega}$.

Proof of Proposition 2: We know that,

$$\begin{aligned}\mathbb{E}_\omega[f(a^*, s^*)] &= \mu\frac{\gamma\overline{\omega}^2\underline{\omega}}{\beta^2} + (1-\mu)\frac{\gamma\overline{\omega}\underline{\omega}^2}{\beta^2} \\ &= \gamma\overline{\omega}\underline{\omega}\left(\frac{\hat{\omega}}{\beta^2}\right)\end{aligned}$$

Now taking the derivative of $\mathbb{E}_\omega[f(a^*, s^*)]$ with respect to μ, we obtain,

$$\frac{\partial\mathbb{E}_\omega[f(a^*, s^*)]}{\partial\mu} = \gamma\overline{\omega}\underline{\omega}\left(\frac{\frac{\partial\hat{\omega}}{\partial\mu}\beta^2 - 2\beta\hat{\omega}\frac{\partial\beta}{\partial\mu}}{\beta^4}\right)$$

We know that $\frac{\partial\hat{\omega}}{\partial\mu} = \overline{\omega} - \underline{\omega}$. Therefore, the numerator of the term in parenthesis is negative if

$$(\overline{\omega} - \underline{\omega})\beta(\beta - 2\gamma^2\hat{\omega}) < 0 \tag{4}$$

which is true whenever

$$\gamma > \sqrt{\frac{\overline{\omega}\underline{\omega}}{\hat{\omega}}} \equiv \underline{\gamma},$$

as required. ■

Proof of Proposition 3: We will prove each part separately.

Consider first the case of a high-capacity terrorist group (i.e. $\omega = \underline{\omega}$). We have that, in equilibrium, the expected utility of a high-capacity group is given by

$$\begin{aligned}\mathbb{E}[U_T(\underline{\omega})] &= \gamma f(\underline{a}^*, s^*) - \frac{\underline{\omega} a^{*2}}{2} \\ &= \frac{2\gamma^2\overline{\omega}^2\underline{\omega} - \gamma^2\underline{\omega}\overline{\omega}^2}{2\beta^2} \\ &= \frac{\gamma^2\underline{\omega}\overline{\omega}^2}{2\beta^2}\end{aligned} \tag{5}$$

Similarly, for the case of a low-capacity terrorist group (i.e. $\omega = \overline{\omega}$), we have that, in equilibrium, its expected utility is given by

$$\begin{aligned}\mathbb{E}[U_T(\overline{\omega})] &= \gamma f(\underline{a}^*, s^*) - \frac{\underline{\omega} a^{*2}}{2} \\ &= \frac{\gamma^2\underline{\omega}^2\overline{\omega}}{2\beta^2}\end{aligned} \tag{6}$$

It is immediate to see that the only term that depends on μ is $\hat{\omega} = \mu\overline{\omega}+(1-\mu)\underline{\omega}$ which enters both $\mathbb{E}[U_T(\underline{\omega})]$ and $\mathbb{E}[U_T(\overline{\omega})]$ only at the denominator. Therefore, we have $\frac{\partial\mathbb{E}[U_T(\underline{\omega})]}{\partial\mu} < 0$ and $\frac{\partial\mathbb{E}[U_T(\overline{\omega})]}{\partial\mu} < 0$, as required.

Part (ii). The ex-ante expected utility for G is given by

$$\mathbb{E}[U_G] \equiv \mu\mathbb{E}[U_G|\underline{\omega}] + (1-\mu)\mathbb{E}[U_G|\overline{\omega}] \tag{7}$$

Differentiating the expression above with respect to μ we obtain

$$\begin{aligned}
\frac{\partial \mathbb{E}[U_G]}{\partial \mu} &= \mathbb{E}[U_G|\underline{\omega}] + \mu \frac{\partial \mathbb{E}[U_G|\underline{\omega}]}{\partial \mu} - \mathbb{E}[U_G|\overline{\omega}] + (1-\mu)\frac{\partial \mathbb{E}[U_G|\overline{\omega}]}{\partial \mu} \\
&= \frac{-2\gamma^2\overline{\omega}^2\underline{\omega} - \gamma^4\hat{\omega}^2}{2\beta^2} + \mu \frac{4\gamma^4\underline{\omega}\overline{\omega}\Delta(2\overline{\omega}-\hat{\omega})}{4\beta^3} \\
&- \frac{-2\gamma^2\overline{\omega}\underline{\omega}^2 - \gamma^4\hat{\omega}^2}{2\beta^2} + (1-\mu)\frac{4\gamma^4\underline{\omega}\overline{\omega}\Delta(2\underline{\omega}-\hat{\omega})}{4\beta^3} \\
&= -\frac{-2\gamma^2\overline{\omega}\underline{\omega}\Delta}{2\beta^2} + \frac{4\gamma^4\underline{\omega}\overline{\omega}\Delta\hat{\omega}}{4\beta^3} \\
&= \frac{4\gamma^4\underline{\omega}\overline{\omega}\Delta\hat{\omega} - 4\gamma^2\underline{\omega}\overline{\omega}\Delta\hat{\omega}\beta}{4\beta^3} \\
&= \frac{4\gamma^2\underline{\omega}\overline{\omega}\Delta(\gamma^2\hat{\omega}-\beta)}{4\beta^3} < 0
\end{aligned} \tag{8}$$

where the last inequality follows from the fact that $\beta \equiv \gamma^2\hat{\omega} + \underline{\omega}\overline{\omega}$. ∎

To clarify the role that μ has in the results, we will present a model in which there is no uncertainty and the comparative static is performed on the capacity of the terrorist group, and then we will restate the results in the main text to show how the model with incomplete offers insights that cannot be obtained in a model with complete information.

Suppose first that T's capacity is common knowledge, and it is equal to ω. Maintaining everything else the same, it is immediate to see that

$$s^* = \frac{\gamma^2}{\gamma^2 + \omega}$$

and

$$a^* = \frac{\gamma}{\gamma^2 + \omega}$$

Therefore the equilibrium probability of a terrorist attack is equal to

$$f(a^*, s^*) = a^*(1-s^*) = \frac{\gamma}{\gamma^2+\omega} \cdot \frac{\omega}{\gamma^2+\omega} = \frac{\gamma\omega}{(\gamma^2+\omega)2}$$

From here, taking the derivative with respect to ω, we have

$$\frac{\partial f(a^*, s^*)}{\partial \omega} = \frac{\gamma(\gamma^2+\omega)^2 - 2\gamma\omega}{(\gamma^2+\omega)} = \frac{\gamma^3 - \gamma\omega}{(\gamma^2+\omega)^4}$$

We obtain that $\frac{\partial f(a^*,s^*)}{\partial \omega}$ is negative whenever $\gamma < \sqrt{\omega}$.

Prima facie, it has a similar intuition to the one expressed in the main text: as the terrorist group becomes weaker (i.e. ω increases), the probability of a terrorist attack increases.

From the proof of Proposition 2, we know that $\frac{\partial f(a^*,s^*)}{\partial \mu} < 0$ if (**??**) holds, and that is if

$$(\overline{\omega} - \underline{\omega})\beta(\beta - 2\gamma^2\hat{\omega}) < 0.$$

Since $(\overline{\omega} - \underline{\omega})\beta > 0$, the sign depends on

$$(\beta - 2\gamma^2\hat{\omega}) = \overline{\omega}\underline{\omega} - \gamma^2\hat{\omega}.$$

Manipulating further this expression we obtain

$$\overline{\omega}\underline{\omega} - \gamma^2\hat{\omega} = \overline{\omega}\underline{\omega} - \gamma^2(\mu\overline{\omega} + (1-\mu)\underline{\omega}) = \overline{\omega}\underline{\omega} - \gamma^2\Big(\mu(\overline{\omega} - \underline{\omega}) + \underline{\omega}\Big)$$

This expression is negative if

$$\mu > \frac{\overline{\omega}\underline{\omega}}{\gamma^2\Big(\mu(\overline{\omega} - \underline{\omega})\Big)} - \frac{\underline{\omega}}{\Big(\mu(\overline{\omega} - \underline{\omega})\Big)} \equiv \mu^*$$

Simple algebra shows that if $\gamma > \sqrt{\overline{\omega}}$, then $\mu^* < 0$. This implies that if $\gamma > \sqrt{\overline{\omega}}$, then $\frac{\partial f(a^*,s^*)}{\partial \mu} < 0$ for all values of μ, since $\mu > 0 > \mu^*$. If instead $\gamma < \sqrt{\overline{\omega}}$, $\mu^* > 0$. Therefore, if the stakes of prevention are not too high, the probability of a terrorist attack is non-monotonic in μ: it is increasing for values of μ below μ^*, and it is decreasing for values of

μ above μ^*. This result cannot be obtained in a simple model with complete information, since it specifies how the probability of a terrorist attack varies with μ.

Appendix B

Suppose that G moves before T, and that G's choice, s, is observable. For any level of surveillance s, the optimal level of terrorist activities for a group of capacity $i \in \{L, H\}$ is given by the expressions in the main text, that is,

$$\overline{a}^*(s) = \frac{\gamma(1-s)}{\overline{\omega}} \qquad \textit{and} \qquad \underline{a}^*(s) = \frac{\gamma(1-s)}{\underline{\omega}}, \tag{9}$$

Consider now G's choice. In this case, G internalizes how his choice affects T's choice afterwards. Therefore, G's problem becomes,

$$\max_{s\in[0,1]} -\gamma\Big[\mu\frac{\gamma(1-s)^2}{\underline{\omega}} + (1-\mu)\frac{\gamma(1-s)^2}{\overline{\omega}}\Big] - \frac{s^2}{2} \tag{10}$$

Accordingly, the optimal level of surveillance solves the following first order condition

$$\gamma\Big[2\mu\overline{\omega}\gamma(1-s) + (1-\mu)\underline{\omega}\gamma(1-s)\Big] = s\underline{\omega}\overline{\omega} \tag{11}$$

Rearranging and solving for s we obtain

$$s^* = \frac{2\gamma^2\hat{\omega}}{\lambda}. \tag{12}$$

where $\hat{\omega} \equiv \mu\overline{\omega} + (1-\mu)\underline{\omega}$ and $\lambda \equiv 2\gamma^2\hat{\omega} + \underline{\omega}\overline{\omega}$. Finally, plugging s^* into (**??**) we obtain,

$$\overline{a}^* = \frac{\gamma\overline{\omega}}{\lambda} \qquad \textit{and} \qquad \underline{a}^* = \frac{\gamma\underline{\omega}}{\lambda}. \tag{13}$$

We know that,

$$\begin{aligned}\mathbb{E}_\omega[f(a^*,s^*)] &= \mu\frac{\gamma\overline{\omega}^2\underline{\omega}}{\lambda^2} + (1-\mu)\frac{\gamma\overline{\omega}\underline{\omega}^2}{\lambda^2} \\ &= \gamma\overline{\omega}\underline{\omega}\left(\frac{\hat{\omega}}{\lambda^2}\right)\end{aligned}$$

Now taking the derivative of $\mathbb{E}_\omega[f(a^*,s^*)]$ with respect to μ, we obtain,

$$\frac{\partial\mathbb{E}_\omega[f(a^*,s^*)]}{\partial\mu} = \gamma\overline{\omega}\underline{\omega}\left[\frac{(\overline{\omega}-\underline{\omega})\lambda^2 - 4\lambda\hat{\omega}\gamma^2(\overline{\omega}-\underline{\omega})}{\lambda^4}\right]$$

which after simplifying becomes

$$\frac{\partial\mathbb{E}_\omega[f(a^*,s^*)]}{\partial\mu} = \gamma\overline{\omega}\underline{\omega}\left[\frac{(\overline{\omega}-\underline{\omega})(\lambda - 4\hat{\omega}\gamma^2)}{\lambda^3}\right]$$

The numerator of the term in square brackets is negative if

$$2\gamma^2\hat{\omega} + \underline{\omega}\overline{\omega} - 4\gamma^2\hat{\omega} < 0$$

which is true whenever

$$\gamma > \sqrt{\frac{\overline{\omega}\underline{\omega}}{2\hat{\omega}}} \equiv \underline{\gamma}^\dagger$$

This shows that the main result in the main text is robust to assuming that G moves before T, and s is observable.

Appendix C

In this section, we present a version of the model where the government is uncertain about how the terrorists value a successful attack, and not about the cost of conducting terrorist activities. Specifically, suppose that T's value from a successful attack is $\gamma \in \{\underline{\gamma}, \overline{\gamma}\}$, with $\overline{\gamma} > \underline{\gamma}$. Suppose further that μ is the probability with which G is facing a group that values a terrorist attack at $\gamma = \overline{\gamma}$. For ease of exposition, let us call such groups *extremists*, and the groups that value a terrorist attack at $\gamma = \underline{\gamma}$ *moderates*. Finally, assume that the cost G pays following a successful attack is given by π.

For any level of surveillance s, the optimal level of terrorist activities for a group that values a successful attack at $\gamma \in \{\underline{\gamma}, \overline{\gamma}\}$ is given by,

$$\overline{a}^*(s) = \frac{\overline{\gamma}(1-s)}{\omega} \qquad \textit{and} \qquad \underline{a}^*(s) = \frac{\underline{\gamma}(1-s)}{\omega}, \tag{14}$$

Consider now G's problem. Call $\hat{a} = \mu\overline{a} + (1-\mu)\underline{a}$. We have,

$$\max_{s\in[0,1]} -\pi\hat{a}(1-s) - \frac{s^2}{2} \tag{15}$$

Accordingly, the optimal level of surveillance solves the following first order condition

$$s^* = \pi\hat{a} \tag{16}$$

By substitution, we obtain,

$$\omega s^* = \pi(1-s^*)\Big(\mu\overline{\gamma} + (1-\mu)\underline{\gamma}\Big) \tag{17}$$

Rearranging and solving for s we obtain

$$s^* = \frac{\pi\hat{\gamma}}{\pi\hat{\gamma} + \omega}. \tag{18}$$

where $\hat{\gamma} \equiv \mu\overline{\gamma} + (1-\mu)\underline{\gamma}$. Finally, plugging s^* into (**??**) we obtain,

$$\overline{a}^* = \frac{\overline{\gamma}}{\pi\hat{\gamma} + \omega} \qquad \textit{and} \qquad \underline{a}^* = \frac{\underline{\gamma}}{\pi\hat{\gamma} + \omega}. \tag{19}$$

It is easy to see that an increase in μ leads to a decrease in $\overline{a}^*$ and $\underline{a}^*$, since $\frac{\partial \hat{\gamma}}{\partial \mu} = \overline{\gamma} - \underline{\gamma} > 0$.

Moreover, an increase μ leads to an increase in s^*,

$$\frac{\partial s^*}{\partial \mu} = \frac{\pi \frac{\partial \hat{\gamma}}{\partial \mu}(\pi\hat{\gamma} + \omega) - \pi^2 \hat{\gamma} \frac{\partial \hat{\gamma}}{\partial \mu}}{(\pi\hat{\gamma} + \omega)^2} > 0 \tag{20}$$

We know that,

$$\begin{aligned}
\mathbb{E}_\omega[f(a^*, s^*)] &= (\mu\overline{a}^* + (1-\mu)\underline{a}^*)(1 - s^*) \\
&= \frac{\hat{\gamma}}{\pi\hat{\gamma} + \omega} \frac{\omega}{\pi\hat{\gamma} + \omega} \\
&= \frac{\omega\hat{\gamma}}{(\pi\hat{\gamma} + \omega)^2}
\end{aligned}$$

Now taking the derivative of $\mathbb{E}_\omega[f(a^*, s^*)]$ with respect to μ, we obtain,

$$\frac{\partial \mathbb{E}_\omega[f(a^*, s^*)]}{\partial \mu} = \frac{\omega \frac{\partial \hat{\gamma}}{\partial \mu}(\pi\hat{\gamma} + \omega)^2 - 2\omega\hat{\gamma}(\pi\hat{\gamma} + \omega)\pi \frac{\partial \hat{\gamma}}{\partial \mu}}{(\pi\hat{\gamma} + \omega)^4}$$

which after simplifying becomes

$$\frac{\partial \mathbb{E}_\omega[f(a^*, s^*)]}{\partial \mu} = \frac{\omega \frac{\partial \hat{\gamma}}{\partial \mu}(\pi\hat{\gamma} + \omega - 2\pi\hat{\gamma})}{(\pi\hat{\gamma} + \omega)^3}$$

The numerator is negative if

$$\gamma - \pi\hat{\gamma} < 0$$

which is true whenever

$$\hat{\gamma} > \frac{\omega}{\pi}$$

or if

$$\pi > \frac{\omega}{\hat{\gamma}} \equiv \pi^*$$

This shows that the main result in the main text is robust to assuming that G is uncertain about T's value of conducting a successful attack rather than T's cost of conducting terrorist activities: if the stakes of terrorism prevention for G are high enough (i.e. $\pi > \pi^*$), an increase in μ reduces the expected probability of a terrorist attack.

Appendix D

In this section, we present a version of the model where the cost the government pays after a successful attack is different from the benefit derived by the terrorist group. Specifically, assume that T's benefit is given by γ, while G's cost is given by π.

For any level of surveillance s, the optimal level of terrorist activities for a group of capacity $i \in \{L, H\}$ is given by the expressions in the main text, that is,

$$\overline{a}^*(s) = \frac{\gamma(1-s)}{\overline{\omega}} \qquad \textit{and} \qquad \underline{a}^*(s) = \frac{\gamma(1-s)}{\underline{\omega}}, \tag{21}$$

Move now to G's choice. The optimal level of surveillance solves the following first order condition

$$s^* = \pi\left(\mu\frac{\gamma(1-s)}{\underline{\omega}} + (1-\mu)\frac{\gamma(1-s)}{\overline{\omega}}\right) \tag{22}$$

Rearranging and solving for s we obtain

$$s^* = \frac{\pi\gamma\hat{\omega}}{\overline{\omega}\underline{\omega} + \pi\gamma\hat{\omega}}. \tag{23}$$

where $\hat{\omega} \equiv \mu\overline{\omega} + (1-\mu)\underline{\omega}$. Finally, by substitution we obtain,

$$\overline{a}^* = \frac{\gamma\underline{\omega}}{\overline{\omega}\underline{\omega} + \pi\gamma\hat{\omega}} \qquad \textit{and} \qquad \underline{a}^* = \frac{\gamma\overline{\omega}}{\overline{\omega}\underline{\omega} + \pi\gamma\hat{\omega}}. \tag{24}$$

We know that,

$$\mathbb{E}_\omega[f(a^*,s^*)] = \left(\mu\frac{\gamma\overline{\omega}}{\overline{\omega}\underline{\omega}+\pi\gamma\hat{\omega}} + (1-\mu)\frac{\gamma\underline{\omega}}{\overline{\omega}\underline{\omega}+\pi\gamma\hat{\omega}}\right)\frac{\overline{\omega}\underline{\omega}}{\overline{\omega}\underline{\omega}+\pi\gamma\hat{\omega}}$$
$$= \left(\frac{\gamma\overline{\omega}\underline{\omega}\hat{\omega}}{(\overline{\omega}\underline{\omega}+\pi\gamma\hat{\omega})^2}\right)$$

Now taking the derivative of $\mathbb{E}_\omega[f(a^*,s^*)]$ with respect to μ, we obtain,

$$\frac{\partial\mathbb{E}_\omega[f(a^*,s^*)]}{\partial\mu} = \gamma\overline{\omega}\underline{\omega}\left[\frac{(\overline{\omega}-\underline{\omega})(\overline{\omega}\underline{\omega}+\pi\gamma\hat{\omega})^2 - 2(\overline{\omega}\underline{\omega}+\pi\gamma\hat{\omega})\hat{\omega}\gamma\pi(\overline{\omega}-\underline{\omega})}{(\overline{\omega}\underline{\omega}+\pi\gamma\hat{\omega})^4}\right]$$

which after simplifying becomes

$$\frac{\partial\mathbb{E}_\omega[f(a^*,s^*)]}{\partial\mu} = \gamma\overline{\omega}\underline{\omega}\left[\frac{(\overline{\omega}-\underline{\omega})(\overline{\omega}\underline{\omega}+\pi\gamma\hat{\omega}-2\hat{\omega}\gamma\pi)}{(\overline{\omega}\underline{\omega}+\pi\gamma\hat{\omega})^3}\right]$$

The numerator of the term in square brackets is negative if

$$\overline{\omega}\underline{\omega} - \pi\gamma\hat{\omega} < 0$$

which is true whenever

$$\gamma > \frac{\overline{\omega}\underline{\omega}}{\pi\hat{\omega}}$$

or when

$$\pi > \frac{\overline{\omega}\underline{\omega}}{\gamma\hat{\omega}}$$

This shows that the main result in the main text is robust to assuming that G's cost from terrorism and T's benefit are not equal.

Appendix E

In this section, we present a version of the model where we try to understand how different types of governments are vulnerable to terrorist attack, and how the government's capacity to enact anti-terrorism operations affect the impact that μ has on the expected probability of a terrorist attack. To do so, suppose the cost the government pays from anti-terrorism activities is scaled by ψ, which captures government capacity, concern for civil liberties etc.

For any level of surveillance s, the optimal level of terrorist activities for a group of capacity $i \in \{L, H\}$ is given by the expressions in the main text, that is,

$$\overline{a}^*(s) = \frac{\gamma(1-s)}{\overline{\omega}} \qquad \textit{and} \qquad \underline{a}^*(s) = \frac{\gamma(1-s)}{\underline{\omega}}, \tag{25}$$

Move now to G's choice. The optimal level of surveillance solves the following first order condition

$$\psi s^* = \pi\left(\mu\frac{\gamma(1-s)}{\underline{\omega}} + (1-\mu)\frac{\gamma(1-s)}{\overline{\omega}}\right) \tag{26}$$

Rearranging and solving for s we obtain

$$s^* = \frac{\pi\gamma\hat{\omega}}{\psi\overline{\omega}\underline{\omega} + \pi\gamma\hat{\omega}}. \tag{27}$$

where $\hat{\omega} \equiv \mu\overline{\omega} + (1-\mu)\underline{\omega}$. Finally, by substitution we obtain,

$$\overline{a}^* = \frac{\gamma\psi\underline{\omega}}{\psi\overline{\omega}\underline{\omega} + \pi\gamma\hat{\omega}} \qquad \textit{and} \qquad \underline{a}^* = \frac{\psi\gamma\overline{\omega}}{\psi\overline{\omega}\underline{\omega} + \pi\gamma\hat{\omega}}. \tag{28}$$

We know that,

$$\begin{aligned}
\mathbb{E}_\omega[f(a^*, s^*)] &= \left(\mu\frac{\gamma\psi\overline{\omega}}{\psi\overline{\omega}\underline{\omega} + \gamma^2\hat{\omega}} + (1-\mu)\frac{\gamma\psi\underline{\omega}}{\psi\overline{\omega}\underline{\omega} + \gamma^2\hat{\omega}}\right)\frac{\psi\overline{\omega}\underline{\omega}}{\psi\overline{\omega}\underline{\omega} + \gamma^2\hat{\omega}} \\
&= \left(\frac{\gamma\psi^2\overline{\omega}\underline{\omega}\hat{\omega}}{(\psi\overline{\omega}\underline{\omega} + \gamma^2\hat{\omega})^2}\right)
\end{aligned}$$

Now taking the derivative of $\mathbb{E}_\omega[f(a^*, s^*)]$ with respect to μ, we obtain,

$$\frac{\partial \mathbb{E}_\omega[f(a^*, s^*)]}{\partial \mu} = \gamma\psi^2\overline{\omega}\underline{\omega}\left[\frac{(\overline{\omega}-\underline{\omega})(\psi\overline{\omega}\underline{\omega}+\gamma^2\hat{\omega})^2 - 2(\psi\overline{\omega}\underline{\omega}+\gamma^2\hat{\omega})\hat{\omega}\gamma^2(\overline{\omega}-\underline{\omega})}{(\psi\overline{\omega}\underline{\omega}+\gamma^2\hat{\omega})^4}\right]$$

which after simplifying becomes

$$\frac{\partial \mathbb{E}_\omega[f(a^*, s^*)]}{\partial \mu} = \gamma\psi^2\overline{\omega}\underline{\omega}\left[\frac{(\overline{\omega}-\underline{\omega})(\psi\overline{\omega}\underline{\omega}+\gamma^2\hat{\omega}-2\hat{\omega}\gamma^2)}{(\psi\overline{\omega}\underline{\omega}+\gamma^2\hat{\omega})^3}\right]$$

The numerator of the term in square brackets is negative if

$$\psi\overline{\omega}\underline{\omega}-\gamma^2\hat{\omega}<0$$

which is true whenever

$$\gamma > \sqrt{\frac{\psi\overline{\omega}\underline{\omega}}{\hat{\omega}}}$$

This mirrors the main result in the main text. Let us move to see how government capacity affects the expected probability of a terrorist attack, and how it affects the impact of μ on the expected probability of a terrorist attack.

To start with, let us assess how the optimal actions are affected by an increase in ψ. It is immediate to see that s^* is decreasing in ψ, as it only appears at the denominator. This is intuitive: as government capacity goes down, meaning that counterterrorism measures and operations are more costly, the government will choose a lower level of s. It is also easy to verify that $\underline{a}^*$ and $\overline{a}^*$ are increasing in ψ. Intuitively, as the government faces increasing costs from counterterrorism operations, the terrorists will become more daring in attempting to conduct a terrorist attack.

Let us move on to the effect of ψ on the expected probability of a terrorist attack. Taking the derivative of $\mathbb{E}_\omega[f(a^*, s^*)]$ with respect to ψ, we obtain,

$$\frac{\partial \mathbb{E}_\omega[f(a^*, s^*)]}{\partial \psi} = \frac{2\gamma\psi\overline{\omega}\underline{\omega}\hat{\omega}(\psi\overline{\omega}\underline{\omega} + \gamma^2\hat{\omega})^2 - 2(\psi\overline{\omega}\underline{\omega} + \gamma^2\hat{\omega})\hat{\omega}\gamma\psi^2\overline{\omega}^2\underline{\omega}^2}{(\overline{\omega}\underline{\omega} + \gamma^2\hat{\omega})^4}$$

which after simplifying becomes

$$\frac{\partial \mathbb{E}_\omega[f(a^*, s^*)]}{\partial \psi} = \frac{2\gamma^3\psi\overline{\omega}\underline{\omega}\hat{\omega}^2}{(\psi\overline{\omega}\underline{\omega} + \gamma^2\hat{\omega})^3}$$

It is easy to see that both the numerator and the denominator are positive, thus implying that an increase in ψ leads to an increase in the expected probability of a terrorist attack.

Finally, let us see how an increase in ψ affects the impact of μ on the expected probability of a terrorist attack.

Taking the derivative of $\frac{\partial \mathbb{E}_\omega[f(a^*, s^*)]}{\partial \psi}$ with respect to μ, we obtain,

$$\frac{\partial^2 \mathbb{E}_\omega[f(a^*, s^*)]}{\partial \psi \partial \mu} = \frac{4\gamma^3\psi\overline{\omega}\underline{\omega}\hat{\omega}(\overline{\omega} - \underline{\omega})(\psi\overline{\omega}\underline{\omega} + \gamma^2\hat{\omega})^3 - 6\gamma^5\psi\underline{\omega}\overline{\omega}\hat{\omega}^2(\overline{\omega} - \underline{\omega})(\psi\overline{\omega}\underline{\omega} + \gamma^2\hat{\omega})^2}{(\psi\overline{\omega}\underline{\omega} + \gamma^2\hat{\omega})^6}$$

After rearranging we obtain,

$$\frac{\partial^2 \mathbb{E}_\omega[f(a^*, s^*)]}{\partial \psi \partial \mu} = \frac{2\gamma^3\psi\overline{\omega}\underline{\omega}\hat{\omega}(\overline{\omega} - \underline{\omega})\Big(2\psi\overline{\omega}\underline{\omega} + 2\gamma^2\hat{\omega} - 3\gamma^2\hat{\omega}\Big)}{(\psi\overline{\omega}\underline{\omega} + \gamma^2\hat{\omega})^4}$$

The denominator is positive. For the numerator to be positive, it has to be that

$$2\psi\overline{\omega}\underline{\omega} > \gamma^2\hat{\omega}$$

which is true whenever

$$\gamma < \sqrt{\frac{2\psi\overline{\omega}\underline{\omega}}{\hat{\omega}}}$$

Recall that $\frac{\partial \mathbb{E}_\omega[f(a^*, s^*)]}{\partial \mu} < 0$ if $\gamma > \sqrt{\frac{\psi\overline{\omega}\underline{\omega}}{\hat{\omega}}}$

For convenience call $\gamma^{\ddagger} = \sqrt{\frac{2\psi\varpi\omega}{\hat{\omega}}}$ and $\gamma^{\dagger} = \sqrt{\frac{\psi\varpi\omega}{\hat{\omega}}}$. Clearly $\gamma^{\ddagger} > \gamma^{\dagger}$.

Therefore we can state the following result.

Proposition 4 *(i) If $\gamma < \gamma^{\dagger}$, an increase in μ leads to an increase in the expected probability of a terrorist attack, and this effect is increasing in ψ;*

(ii) if $\gamma \in (\gamma^{\dagger}, \gamma^{\ddagger})$, an increase in μ leads to a decrease in the expected probability of a terrorist attack, and this effect is increasing in ψ;

(iii) if $\gamma > \gamma^{\ddagger}$, an increase in μ leads to a decrease in the expected probability of a terrorist attack, and this effect is decreasing in ψ.

When the stakes of terrorism prevention are low enough, the more the government is likely to face a strong group, the higher the expected probability of a terrorist attack is. This effect is reduced when the government has more capacity to conduct ant-terrorism activities. For intermediate stakes of terrorism prevention, the more the government is likely to face a strong group, the lower the expected probability of a terrorist attack is. As the government becomes more capable to conduct anti-terrorist activities, this security-enhancing effect becomes even larger. Finally, when the stakes of terrorism prevention are high, the effect of μ on the expected probability of a terrorist attack is the same as in the intermediate case. However, the more capable the government is, the lower this security-enhancing effect is.

Appendix F

In this section we present a more general version of our model, where we do not impose a specific functional form for the probability of a terrorist attack and for the cost functions. A government (G) and a terrorist group (T) interact strategically. G chooses a level of counterterrorism activities $s \in \mathbb{R}_+$ at a cost $c(s)$, which is and increasing and convex function of s. T chooses a level of effort in planning terrorist acts, $a \in \mathbb{R}_+$, at a cost $k(a, \omega)$, that is increasing and convex in a. The parameter $\omega \in \mathbb{R}_+$ captures T's capacity: the higher ω is, the easier it is for T to attract funding, recruit operatives.

G does not know T's capacity with certainty. To model this uncertainty, we assume that T's capacity, ω can be either high or low, that is $\omega \in \{\omega_H, \omega_L\}$ with $\omega_H > \omega_L$. The cost of conducting terrorist activities is given by the function $k(\omega, a)$. For any level of terrorist activities a, we have that $\frac{\partial k(\omega,a)}{\partial \omega} < 0$ and $\frac{\partial^2 k(\omega,a)}{\partial a \partial \omega} < 0$, meaning that a group with higher capacity has a lower cost and a lower marginal cost of conducting terrorist activities. The parameter ω is T's private information, while G's prior that T has high capacity is $Pr(\omega = \omega_H) = \mu \in [0, 1]$. As such, a higher μ reflects G increasing concern about facing a high-capacity T.

G and T choose their action simultaneously. Given the levels of government surveillance and terrorist activities, a terrorist attack happens with probability $f(a, s)$, where the function f is in increasing ad concave in a, decreasing and convex in s, with $\frac{\partial^2 f(a,s)}{\partial a \partial s} < 0$ for all (a, s). The assumptions on the first derivatives and on the cross-partial derivative are intuitive. All else equal, the probability of a successful attack is decreasing in the level of surveillance and increasing in the effort exerted by terrorists. The assumption on the cross-partial derivative (i.e. $\frac{\partial^2 f(a,s)}{\partial a \partial s} < 0$) formalizes the intuitive notion that the effectiveness of the terrorists' effort to successfully execute a terror plot is higher when the government's surveillance to detect traces of terrorist activity is lower.

Given these specifications, G's expected payoff is given by

$$U_G(a, s) = -\gamma f(a, s) - c(s), \tag{29}$$

and T's expected payoff is given by

$$U_T(a, s, \omega) = \gamma f(a, s) - k(\omega, a). \tag{30}$$

We solve for the Bayesian Nash equilibrium of this game, henceforth *equilibrium*. Informally, an equilibrium in this game is given by 1) a surveillance strategy for G and 2) an effort strategy for each type of T.

Consider first T's choice. For any level of surveillance s, the optimal level of terrorist activities for a group of capacity $i \in \{L, H\}$ is characterized by,

$$\gamma \frac{\partial f(a,s)}{\partial a} - \frac{\partial k(\omega_i, a)}{\partial a} = 0. \tag{31}$$

Since $\frac{\partial k(\omega,a)}{\partial a}$ in decreasing in ω, we have that,

$$a_H^*(s) > a_L^*(s). \tag{32}$$

The best response functions of the high-capacity and low-capacity T are *decreasing* in the level of surveillance chosen by G. An increase in surveillance effort reduces the probability of executing a terrorist attack. Anticipating this, T prefers to scale down its effort into planning terror acts.

Consider now G's choice. For any level of terrorist activities a, the optimal level of surveillance is characterized by,

$$\gamma \frac{\partial \mathbb{E}_\omega[f(a,s)]}{\partial s} - c'(s) = 0. \tag{33}$$

which becomes

$$\gamma \left(\mu \frac{\partial f(a_H, s)}{\partial s} + (1-\mu) \frac{\partial f(a_L, s)}{\partial s} \right) - c'(s) = 0. \tag{34}$$

G's best response function is *increasing* in level of effort towards the planning of terrorist activities chosen by the two types of T. As terrorists become more menacing for G's security, G becomes more willing to sustain the cost of additional anti-terrorism measures for the sake of terrorism prevention.

Proposition 5 *There exists a unique equilibrium of the game.*

Proof: Notice that the best responses for both types of T are decreasing in s. This implies that for a given μ, $\hat{a}(s) = \mu a_H(s) + (1-\mu)a_L(s)$ is also decreasing in s. Moreover, the best response of G is increasing in $\hat{a}$ and hence in a_H and in a_L. This means that G's best response function and the convex combination of the best response functions of the two types of T cross once. This characterizes $(\hat{a}^*, s^*)$. Then, the equilibrium actions for the two types of T are given by $a_H^* = a_H(s^*)$ and $a_L^* = a_L(s^*)$, with $\hat{a}^* = \mu a_H^* + (1-\mu)a_L^*$. This completes the argument. ■

Once we have pinned down the equilibrium behavior of the players, we can start assessing how their equilibrium behavior and the outcome of the conflict change in G's belief about T having high capacity. First off, let us assess the impact of μ on the equilibrium actions of G and T.

Proposition 6 *An increase in μ leads to*

(i) an increase in the optimal level of surveillance, s^*;

(ii) a decrease in the optimal level of terrorist activities, a_L^* and a_H^*;

Proof: We know that the unique Bayesian Nash Equilibrium is the solution to the following system of equation:

$$\gamma \frac{\partial f(a_H^*, s^*)}{\partial a} - \frac{\partial k(\omega_H, a_H^*)}{\partial a} = 0. \tag{35}$$

$$\gamma \frac{\partial f(a_L^*, s^*)}{\partial a} - \frac{\partial k(\omega_L, a_L^*)}{\partial a} = 0. \tag{36}$$

$$\gamma \left(\mu \frac{\partial f(a_H^*, s^*)}{\partial s} + (1-\mu) \frac{\partial f(a_L^*, s^*)}{\partial s} \right) - c'(s^*) = 0. \tag{37}$$

Taking the total derivative of (**??**), (**??**), and (**??**) with respect to μ, we obtain

$$\gamma\left(\frac{\partial^2 f(a_H^*, s^*)}{\partial a^2}\frac{da_H^*}{d\mu} + \frac{\partial^2 f(a_H^*, s^*)}{\partial s \partial a}\frac{ds^*}{d\mu}\right) - \frac{\partial^2 k(\omega_H, a_H^*)}{\partial a^2}\frac{da_H^*}{d\mu} = 0 \tag{38}$$

$$\gamma\left(\frac{\partial^2 f(a_L^*, s^*)}{\partial a^2}\frac{da_L^*}{d\mu} + \frac{\partial^2 f(a_L^*, s^*)}{\partial s \partial a}\frac{ds^*}{d\mu}\right) - \frac{\partial^2 k(\omega_L, a_L^*)}{\partial a^2}\frac{da_L^*}{d\mu} = 0 \tag{39}$$

$$\begin{aligned} &-\gamma\left[\frac{\partial f(a_H^*, s^*)}{\partial s} - \frac{\partial f(a_L^*, s^*)}{\partial s} + \mu\left(\frac{\partial^2 f(a_H^*, s^*)}{\partial s^2}\frac{ds^*}{d\mu} + \frac{\partial^2 f(a_H^*, s^*)}{\partial s \partial a}\frac{da_H^*}{d\mu}\right) + \right.\\ &\left. + (1-\mu)\left(\frac{\partial^2 f(a_L^*, s^*)}{\partial s^2}\frac{ds^*}{d\mu} + \frac{\partial^2 f(a_L^*, s)}{\partial s \partial a}\frac{da_L^*}{d\mu}\right) - c''(s^*)\frac{ds^*}{d\mu}\right] = 0, \end{aligned} \tag{40}$$

Call the matrix of coefficients

$$\mathbf{A} = \begin{bmatrix} \gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial a^2} - \frac{\partial^2 k(\omega_H,a_H^*)}{\partial a^2} & 0 & \gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial s \partial a} \\ 0 & \gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial a^2} - \frac{\partial^2 k(\omega_L,a_L^*)}{\partial a^2} & \gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial s \partial a} \\ -\mu\gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial s \partial a} & -(1-\mu)\gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial s \partial a} & -\gamma\left(\mu\frac{\partial^2 f(a_H^*,s^*)}{\partial s^2} + (1-\mu)\frac{\partial^2 f(a_L^*,s^*)}{\partial s^2}\right) - c''(s^*) \end{bmatrix},$$

call the column vector of the variables

$$\mathbf{x} = \begin{bmatrix} \frac{da_H^*}{d\mu} \\ \frac{da_L^*}{d\mu} \\ \frac{ds^*}{d\mu} \end{bmatrix},$$

and finally call

$$\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \gamma\left(\frac{\partial f(a_H^*,s^*)}{\partial s} - \frac{\partial f(a_L^*,s^*)}{\partial s}\right) \end{bmatrix}$$

Hence, we can write the above system of equations in matrix form as,

$$\mathbf{Ax} = \mathbf{b}$$

Now, we have that

$$det(\mathbf{A}) = \left(\gamma\frac{\partial^2 f(a_H^*, s^*)}{\partial a^2} - \frac{\partial^2 k(\omega_H, a_H^*)}{\partial a^2}\right)\left(\gamma\frac{\partial^2 f(a_L^*, s^*)}{\partial a^2} - \frac{\partial^2 k(\omega_L, a_L^*)}{\partial a^2}\right)\Bigg[-c''(s) + \\ -\gamma\left(\mu\frac{\partial^2 f(a_H^*, s^*)}{\partial s^2} + (1-\mu)\frac{\partial^2 f(a_L^*, s^*)}{\partial s^2}\right)\Bigg] + \mu\left(\gamma\frac{\partial^2 f(a_H^*, s^*)}{\partial s \partial a}\right)^2\left(\gamma\frac{\partial^2 f(a_L^*, s^*)}{\partial a^2} - \frac{\partial^2 k(\omega_L, a_L^*)}{\partial a^2}\right) + \\ + (1-\mu)\left(\gamma\frac{\partial^2 f(a_L^*, s^*)}{\partial s \partial a}\right)^2\left(\gamma\frac{\partial^2 f(a_H^*, s^*)}{\partial a^2} - \frac{\partial^2 k(\omega_H, a_H^*)}{\partial a^2}\right)$$

Now, we know that $\frac{\partial^2 f(a_i,s)}{\partial a_i^2} < 0$, $\frac{\partial^2 k(\omega_i,a_i)}{\partial a_i^2} > 0$, $\frac{\partial^2 f(a_i,s)}{\partial s \partial a_i} < 0$, and $\frac{\partial^2 f(a_i,s)}{\partial s^2}$ for all $i \in \{H, L\}$. This means that $det(\mathbf{A}) < 0$. Using Cramer's Rule, we know that

$$\frac{da_H^*}{d\mu} = \frac{\begin{vmatrix} 0 & 0 & \gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial s \partial a} \\ 0 & \gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial a^2} - \frac{\partial^2 k(\omega_L,a_L^*)}{\partial a^2} & \gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial s \partial a} \\ \gamma\left(\frac{\partial f(a_H^*,s^*)}{\partial s} - \frac{\partial f(a_L^*,s^*)}{\partial s}\right) & -\mu\gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial s \partial a} & -\gamma\left(\mu\frac{\partial^2 f(a_H^*,s^*)}{\partial s^2} + (1-\mu)\frac{\partial^2 f(a_L^*,s^*)}{\partial s^2}\right) - c''(s) \end{vmatrix}}{det(\mathbf{A})}$$

$$= \frac{-\gamma\left(\frac{\partial f(a_H^*,s^*)}{\partial s} - \frac{\partial f(a_L^*,s^*)}{\partial s}\right)\left(\gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial a^2} - \frac{\partial^2 k(\omega_L,a_L^*)}{\partial a^2}\right)\gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial s \partial a}}{det(\mathbf{A})} < 0$$

$$\frac{da_L^*}{d\mu} = \frac{\begin{vmatrix} \gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial a^2} - \frac{\partial^2 k(\omega_H,a_H^*)}{\partial a^2} & 0 & \gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial s \partial a} \\ 0 & 0 & \gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial s \partial a} \\ -\mu\gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial s \partial a} & \gamma\left(\frac{\partial f(a_H^*,s^*)}{\partial s} - \frac{\partial f(a_L^*,s^*)}{\partial s}\right) & -\gamma\left(\mu\frac{\partial^2 f(a_H^*,s^*)}{\partial s^2} + (1-\mu)\frac{\partial^2 f(a_L^*,s^*)}{\partial s^2}\right) - c''(s) \end{vmatrix}}{det(\mathbf{A})}$$

$$= \frac{-\gamma\left(\frac{\partial f(a_H^*,s^*)}{\partial s} - \frac{\partial f(a_L^*,s^*)}{\partial s}\right)\left(\gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial a^2} - \frac{\partial^2 k(\omega_H,a_H^*)}{\partial a^2}\right)\gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial s \partial a}}{det(\mathbf{A})} < 0$$

$$\frac{ds^*}{d\mu} = \frac{\begin{vmatrix} \gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial a^2} - \frac{\partial^2 k(\omega_H,a_H^*)}{\partial a^2} & 0 & 0 \\ 0 & \gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial a^2} - \frac{\partial^2 k(\omega_L,a_L^*)}{\partial a^2} & 0 \\ -\mu\gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial s\partial a} & -\mu\gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial s\partial a} & \gamma\left(\frac{\partial f(a_H^*,s^*)}{\partial s} - \frac{\partial f(a_L^*,s^*)}{\partial s}\right) \end{vmatrix}}{det(\mathbf{A})}$$

$$= \frac{\gamma\left(\frac{\partial f(a_H^*,s^*)}{\partial s} - \frac{\partial f(a_L^*,s^*)}{\partial s}\right)\left(\gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial a^2} - \frac{\partial^2 k(\omega_H,a_H^*)}{\partial a^2}\right)\left(\gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial a^2} - \frac{\partial^2 k(\omega_L,a_L^*)}{\partial a^2}\right)}{det(\mathbf{A})} > 0$$

Taken together, these facts prove the result. ■

Let us now assess the effect of μ on the equilibrium probability of a terrorist attack. Denote the equilibrium probability of a terrorist attack when T has capacity $\omega_i \in \{\omega_H, \omega_L\}$ by $f(a_i^*, s^*) \equiv a_i^*(1-s^*)$. Then, the ex-ante equilibrium probability of a terrorist attack is given by,

$$\mathbb{E}_\omega[f(a^*, s^*)] \equiv \mu f(a_H^*, s^*) + (1-\mu) f(a_L^*, s^*), \tag{41}$$

where the subscript ω indicate that the expectation is taken with respect to T's capacity.

Proposition 7 *There exist* Ψ *and* Λ *such that the* ex-ante *equilibrium probability of a terrorist attack,* $\mathbb{E}_\omega[f(a^*, s^*)]$ *is,*

- *decreasing in* μ *for all* μ*, if* $\Lambda > 0$ *and* $\Psi < 0$*;*
- *decreasing in* μ *if* $\mu > \mu^* = \frac{\Psi}{\Lambda}$*, if* $\Lambda > 0$ *and* $\Psi > 0$*;*
- *decreasing in* μ *if* $\mu < \mu^* = \frac{\Psi}{\Lambda}$*, if* $\Lambda < 0$ *and* $\Psi < 0$*;*
- *increasing in* μ *for all* μ*, if* $\Lambda < 0$ *and* $\Psi > 0$*.*

Proof: Taking the total derivative of $\mathbb{E}_\omega[f(a^*, s^*)]$ with respect to μ we obtain

$$f(a_H^*, s^*) - f(a_L^*, s^*) + \mu \frac{df(a_H^*, s^*)}{d\mu} + (1-\mu)\frac{df(a_L^*, s^*)}{d\mu} \tag{42}$$

Now, we have that

$$\frac{df(a_H^*, s^*)}{d\mu} = \frac{\partial f(a_H^*, s^*)}{\partial a}\frac{da_H^*}{d\mu} + \frac{\partial f(a_H^*, s^*)}{\partial s}\frac{ds^*}{d\mu} \tag{43}$$

and that

$$\frac{df(a_L^*, s^*)}{d\mu} = \frac{\partial f(a_L^*, s^*)}{\partial a}\frac{da_L^*}{d\mu} + \frac{\partial f(a_L^*, s^*)}{\partial s}\frac{ds^*}{d\mu} \tag{44}$$

By substitution we obtain total derivative of $\mathbb{E}_\omega[f(a^*, s^*)]$ with respect to μ is

$$f(a_H^*, s^*) - f(a_L^*, s^*) + \mu\left(\frac{\partial f(a_H^*, s^*)}{\partial a}\frac{da_H^*}{d\mu} + \frac{\partial f(a_H^*, s^*)}{\partial s}\frac{ds^*}{d\mu}\right) + (1-\mu)\left(\frac{\partial f(a_L^*, s^*)}{\partial a}\frac{da_L^*}{d\mu} + \frac{\partial f(a_L^*, s^*)}{\partial s}\frac{ds^*}{d\mu}\right) \tag{45}$$

which after rearranging becomes

$$f(a_H^*, s^*) - f(a_L^*, s^*) + \frac{ds^*}{d\mu}\left(\mu\frac{\partial f(a_H^*, s^*)}{\partial s} + (1-\mu)\frac{\partial f(a_L^*, s^*)}{\partial s}\right) + \mu\frac{\partial f(a_H^*, s^*)}{\partial a}\frac{da_H^*}{d\mu} + (1-\mu)\frac{\partial f(a_L^*, s^*)}{\partial a}\frac{da_L^*}{d\mu} \tag{46}$$

The direct effect is given by $f(a_H^*, s^*) - f(a_L^*, s^*)$ and is positive. The indirect effect is given by

$$\frac{ds^*}{d\mu}\left(\mu\frac{\partial f(a_H^*, s^*)}{\partial s} + (1-\mu)\frac{\partial f(a_L^*, s^*)}{\partial s}\right) + \mu\frac{\partial f(a_H^*, s^*)}{\partial a}\frac{da_H^*}{d\mu} + (1-\mu)\frac{\partial f(a_L^*, s^*)}{\partial a}\frac{da_L^*}{d\mu}$$

Let us focus on the indirect effect for now. We can now use the expressions derived in the previous proofs and substitute for $\frac{da_H^*}{d\mu}$, $\frac{da_L^*}{d\mu}$, and $\frac{ds^*}{d\mu}$. Doing this, leads us to

$$\frac{\gamma\left(\frac{\partial f(a_H^*,s^*)}{\partial s}-\frac{\partial f(a_L^*,s^*)}{\partial s}\right)\left(\gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial {a_H^*}^2}-\frac{\partial^2 k(\omega_H,a_H^*)}{\partial {a_H^*}^2}\right)\left(\gamma\frac{\partial^2 f(a_L^*,s)}{\partial {a_L^*}^2}-\frac{\partial^2 k(\omega_L,a_L^*)}{\partial {a_L^*}^2}\right)}{det(\mathbf{A})}\left(\mu\frac{\partial f(a_H^*,s^*)}{\partial s}+(1-\mu)\frac{\partial f(a_L^*,s^*)}{\partial s}\right)+$$
$$-\mu\frac{\partial f(a_H^*,s^*)}{\partial a}\frac{\gamma\left(\frac{\partial f(a_H^*,s^*)}{\partial s}-\frac{\partial f(a_L^*,s^*)}{\partial s}\right)\left(\gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial a^2}-\frac{\partial^2 k(\omega_L,a_L^*)}{\partial a^2}\right)\gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial s\partial a}}{det(\mathbf{A})}+$$
$$-(1-\mu)\frac{\partial f(a_L^*,s^*)}{\partial a}\frac{\gamma\left(\frac{\partial f(a_H^*,s^*)}{\partial s}-\frac{\partial f(a_L^*,s^*)}{\partial s}\right)\left(\gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial a^2}-\frac{\partial^2 k(\omega_H,a_H^*)}{\partial a^2}\right)\gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial s\partial a}}{det(\mathbf{A})} \quad (47)$$

Multiplying by $det(\mathbf{A})$ we have that the total derivative of $\mathbb{E}_\omega[f(a^*,s^*)]$ with respect to μ is equal to

$$\left(f(a_H^*,s^*)-f(a_L^*,s^*)\right)det(\mathbf{A})+\gamma\left(\frac{\partial f(a_H^*,s^*)}{\partial s}-\frac{\partial f(a_L^*,s^*)}{\partial s}\right)\left(\gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial {a_H^*}^2}-\frac{\partial^2 k(\omega_H,a_H^*)}{\partial {a_H^*}^2}\right)\cdot$$
$$\cdot\left(\gamma\frac{\partial^2 f(a_L^*,s)}{\partial {a_L^*}^2}-\frac{\partial^2 k(\omega_L,a_L^*)}{\partial {a_L^*}^2}\right)\left(\mu\frac{\partial f(a_H^*,s^*)}{\partial s}+(1-\mu)\frac{\partial f(a_L^*,s^*)}{\partial s}\right)+$$
$$-\mu\frac{\partial f(a_H^*,s^*)}{\partial a}\gamma\left(\frac{\partial f(a_H^*,s^*)}{\partial s}-\frac{\partial f(a_L^*,s^*)}{\partial s}\right)\left(\gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial a^2}-\frac{\partial^2 k(\omega_L,a_L^*)}{\partial a^2}\right)\gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial s\partial a}+$$
$$-(1-\mu)\frac{\partial f(a_L^*,s^*)}{\partial a}\gamma\left(\frac{\partial f(a_H^*,s^*)}{\partial s}-\frac{\partial f(a_L^*,s^*)}{\partial s}\right)\left(\gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial a^2}-\frac{\partial^2 k(\omega_H,a_H^*)}{\partial a^2}\right)\gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial s\partial a} \quad (48)$$

Recall that

$$det(\mathbf{A})=\left(\gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial a^2}-\frac{\partial^2 k(\omega_H,a_H^*)}{\partial a^2}\right)\left(\gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial a^2}-\frac{\partial^2 k(\omega_L,a_L^*)}{\partial a^2}\right)\Bigg[-c''(s)+$$
$$-\gamma\left(\mu\frac{\partial^2 f(a_H^*,s^*)}{\partial s^2}+(1-\mu)\frac{\partial^2 f(a_L^*,s^*)}{\partial s^2}\right)\Bigg]+\mu\left(\gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial s\partial a}\right)^2\left(\gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial a^2}-\frac{\partial^2 k(\omega_L,a_L^*)}{\partial a^2}\right)+$$
$$+(1-\mu)\left(\gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial s\partial a}\right)^2\left(\gamma\frac{\partial^2 f(a_H^*,s^*)}{\partial a^2}-\frac{\partial^2 k(\omega_H,a_H^*)}{\partial a^2}\right) \quad (49)$$

Moreover, we have that $\frac{d\mathbb{E}_\omega[f(a^*,s^*)]}{d\mu} < 0$ if

$$\textit{Direct Effect} + \frac{\textit{Indirect Effect}}{det(\mathbf{A})} < 0$$

After having multiplied by $det(\mathbf{A})$, which is negative, we have $\frac{\mathbb{E}_\omega[f(a^*,s^*)]}{d\mu} < 0$ if

$$\textit{Direct Effect} \cdot det(\mathbf{A}) + \textit{Indirect Effect} > 0$$

For convenience, define:

- $\Delta_f \equiv f(a_H^*, s^*) - f(a_L^*, s^*)$
- $\theta_H \equiv \gamma \frac{\partial^2 f(a_H^*,s^*)}{\partial a^2} - \frac{\partial^2 k(\omega_H, a_H^*)}{\partial a^2}$
- $\theta_L \equiv \gamma \frac{\partial^2 f(a_L^*,s^*)}{\partial a^2} - \frac{\partial^2 k(\omega_L, a_L^*)}{\partial a^2}$

With these expressions, we can rewrite the direct effect as

$$\begin{aligned} &-\Delta_f \gamma\mu \frac{\partial^2 f(a_H^*, s^*)}{\partial s^2}\theta_H\theta_L - \Delta_f\gamma(1-\mu)\frac{\partial^2 f(a_L^*, s^*)}{\partial s^2}\theta_H\theta_L - c''(s)\Delta_f\theta_H\theta_L + \\ &+\Delta_f\gamma^2\mu\left(\frac{\partial^2 f(a_H^*, s^*)}{\partial s\partial a}\right)^2\theta_L + \Delta_f\gamma^2(1-\mu)\left(\frac{\partial^2 f(a_L^*, s^*)}{\partial s\partial a}\right)^2\theta_H \end{aligned} \tag{50}$$

and the indirect effect as

$$\begin{aligned} &\gamma\left(\mu\frac{\partial f(a_H^*, s^*)}{\partial s} + (1-\mu)\frac{\partial f(a_L^*, s^*)}{\partial s}\right)\gamma\left(\frac{\partial f(a_H^*, s^*)}{\partial s} - \frac{\partial f(a_L^*, s^*)}{\partial s}\right)\theta_H\theta_L + \\ &-\mu\frac{\partial f(a_H^*, s^*)}{\partial a}\gamma^2\theta_L\left(\frac{\partial f(a_H^*, s^*)}{\partial s} - \frac{\partial f(a_L^*, s^*)}{\partial s}\right)\frac{\partial^2 f(a_H^*, s^*)}{\partial s\partial a} \\ &-(1-\mu)\gamma^2\theta_H\frac{\partial f(a_L^*, s^*)}{\partial a}\left(\frac{\partial f(a_H^*, s^*)}{\partial s} - \frac{\partial f(a_L^*, s^*)}{\partial s}\right)\frac{\partial^2 f(a_L^*, s^*)}{\partial s\partial a} \end{aligned} \tag{51}$$

Putting the two effects together we obtain

$$
\begin{aligned}
&-\mu\Delta_f\gamma\theta_H\theta_L\frac{\partial^2 f(a_H^*,s^*)}{\partial s^2}+\mu\Delta_f\gamma\frac{\partial^2 f(a_L^*,s^*)}{\partial s^2}\theta_H\theta_L+\\
&+\mu\Delta_f\gamma\left(\frac{\partial f(a_H^*,s^*)}{\partial s}-\frac{\partial f(a_L^*,s^*)}{\partial s}\right)\frac{\partial f(a_H^*,s^*)}{\partial s}\theta_H\theta_L+\\
&-\mu\Delta_f\gamma\left(\frac{\partial f(a_H^*,s^*)}{\partial s}-\frac{\partial f(a_L^*,s^*)}{\partial s}\right)\frac{\partial f(a_L^*,s^*)}{\partial s}\theta_H\theta_L+\\
&+\mu\gamma^2\theta_L\frac{\partial^2 f(a_H^*,s^*)}{\partial s\partial a}\left[\Delta_f\frac{\partial^2 f(a_H^*,s^*)}{\partial s\partial a}-\frac{\partial f(a_H^*,s^*)}{\partial a}\left(\frac{\partial f(a_H^*,s^*)}{\partial s}-\frac{\partial f(a_L^*,s^*)}{\partial s}\right)\right]+\\
&-\mu\gamma^2\theta_H\frac{\partial^2 f(a_L^*,s^*)}{\partial s\partial a}\left[\Delta_f\frac{\partial^2 f(a_L^*,s^*)}{\partial s\partial a}-\frac{\partial f(a_L^*,s^*)}{\partial a}\left(\frac{\partial f(a_H^*,s^*)}{\partial s}-\frac{\partial f(a_L^*,s^*)}{\partial s}\right)\right]>\\
&\Delta_f\frac{\partial^2 f(a_L^*,s^*)}{\partial s^2}\gamma\theta_H\theta_L-\frac{\partial f(a_L^*,s^*)}{\partial s}\gamma^2\theta_H\theta_L\left(\frac{\partial f(a_H^*,s^*)}{\partial s}-\frac{\partial f(a_L^*,s^*)}{\partial s}\right)+c''(s)\Delta_f\theta_H\theta_L+\\
&-\gamma^2\theta_H\frac{\partial^2 f(a_L^*,s^*)}{\partial s\partial a}\left[\Delta_f\frac{\partial^2 f(a_L^*,s^*)}{\partial s\partial a}-\frac{\partial f(a_L^*,s^*)}{\partial a}\left(\frac{\partial f(a_H^*,s^*)}{\partial s}-\frac{\partial f(a_L^*,s^*)}{\partial s}\right)\right]
\end{aligned}
\tag{52}
$$

Collecting terms we can write the expression above as

$$\mu\Lambda > \Psi \tag{53}$$

Now, if $\Lambda > 0$ and $\Psi > 0$, then $\frac{d\mathbb{E}_\omega[f(a^*,s^*)]}{d\mu} < 0$ if $\mu > \mu^* = \frac{\Psi}{\Lambda}$.

If $\Lambda > 0$ and $\Psi < 0$, then $\frac{d\mathbb{E}_\omega[f(a^*,s^*)]}{d\mu} < 0$ if $\mu > \mu^* = \frac{\Psi}{\Lambda}$, but since $\mu^* < 0$, this is always true.

If $\Lambda < 0$ and $\Psi > 0$, then $\frac{d\mathbb{E}_\omega[f(a^*,s^*)]}{d\mu} < 0$ if $\mu < \mu^* = \frac{\Psi}{\Lambda}$, but since $\mu^* < 0$, this is never true.

If $\Lambda < 0$ and $\Psi < 0$, then $\frac{d\mathbb{E}_\omega[f(a^*,s^*)]}{d\mu} < 0$ if $\mu < \mu^* = \frac{\Psi}{\Lambda}$.

Together, these facts establish the result. ■

Appendix G

In this section, we show that our main results hold in a dynamic (two-period) game in which the government updates about the type of the opponent they faced given the realized outcome and their own investment.

Our main results, propositions $1-3$ investigate how changes in the government's prior belief that it is facing a high capacity terror group, μ, affect the equilibrium actions and the players' expected payoff. All that we require for our results to hold in a dynamic setting with learning is for the updated probability (given the government's prior equilibrium action and the observed outcome of the prior interaction) to be an increasing function in the prior belief μ. We will show below that this is indeed the case and that our main results hold in a dynamic model in which we incorporate a second period, with learning.

Let us consider a two-period dynamic game with the following timing.

- In the first period, G and T choose their actions s^1, $\underline{a}^1$ (if T is high-capacity), and $\overline{a}^1$ (if T is low-capacity).

- The outcome of the interaction in the first period game is a terrorist attack with probability $f(s^1, \underline{a}^1)$ (if T is high-capacity) or with probability $f(s^1, \overline{a}^1)$ (if T is low-capacity).

- Given the outcome of the first period (a terror attack or no terror attack), the government updates its prior μ about T being high-capacity.

- In the second period, G and T choose their actions s^2, $\underline{a}^2$ (if T is high-capacity), and $\overline{a}^2$ (if T is low-capacity).

- The outcome of the interaction in the second period is a terrorist attack with probability $f(s^2, \underline{a}^2)$ (if T is high-capacity) or with probability $f(s^2, \overline{a}^2)$ (if T is low-capacity).

Now let us consider some equilibrium in the first period of the game, $(s^1; \underline{a}^1; \overline{a}^1)$. The outcome of the interaction in the first period game is a terrorist attack with probability $f(s^1, \underline{a}^1)$ (if T is high-capacity) or a terror attack with probability $f(s^1, \overline{a}^1)$ (if T is low-capacity). Given this, let us determine the government's updated probability that T is high-capacity.

If the outcome of the first period is no terror attack, let us denote G's updated belief by $\pi(\mu|A = 0)$ where $A = 0$ denotes the fact that the outcome of the first period was a no terror attack. We then have the following:

$$\pi(\mu|A = 0) = \frac{\mu(1 - f(s^1, \underline{a}^1))}{\mu(1 - f(s^1, \underline{a}^1)) + (1 - \mu)(1 - f(s^1, \overline{a}^1))}.$$

It is easy to check that $\pi(\mu|A = 0)$ is increasing in μ, G's prior belief about T's being high-capacity. This is the case since $\frac{\partial \pi(\mu|A=0)}{\partial \mu} = \frac{(1-f(s^1,\underline{a}^1))(1-f(s^1,\overline{a}^1))}{[\mu(1-f(s^1,\underline{a}^1))+(1-\mu)(1-f(s^1,\overline{a}^1))]^2} > 0$.

If the outcome of the first period is a terror attack, let us denote G's updated belief by $\pi(\mu|A = 1)$ where $A = 1$ denotes the fact that the outcome of the first period was a terror attack. We then have the following:

$$\pi(\mu|A = 1) = \frac{\mu f(s^1, \underline{a}^1)}{\mu f(s^1, \underline{a}^1) + (1 - \mu) f(s^1, \overline{a}^1)}.$$

again, it is easy to check that $\pi(\mu|A = 1)$ is increasing in μ, G's prior belief about T's being high-capacity. This is the case since $\frac{\partial \pi(\mu|A=1)}{\partial \mu} = \frac{f(s^1,\underline{a}^1) f(s^1,\overline{a}^1)}{[\mu f(s^1,\underline{a}^1)+(1-\mu) f(s^1,\overline{a}^1)]^2} > 0$.

Thus, we can see that G's updated belief if the first-period outcome is a terror attack and G's updated belief if the first-period outcome is no terror attack, both $\pi(\mu|A = 1)$ and $\pi(\mu|A = 0)$ are in increasing functions of μ. Our main results pertain to how changes in μ affect the players' equilibrium action and payoff in the stage game. As such, similar results (as proposition $1 - 3$) obtain in a dynamic model in which we incorporate a second period, with learning.

By a similar logic as our analysis, both in the second period (following a terror attack in the first) and the second period (following no terror attack in the first period), there is a unique equilibrium and the equilibrium actions can be obtained by replacing μ in the equilibrium actions characterized in our stage game (expression 3 in our paper) with $\pi(\mu|A = 1)$ or $\pi(\mu|A = 0)$, depending on which second period one considers. More importantly, we can perform similar comparative statics on μ, given that the updated beliefs are increasing in μ, and similar results as stated in propositions $1 - 3$ obtain in the second period, while incorporating learning in this dynamic game.